**Prove that the sum of two even integers is even.**

Restatement in symbols: $\forall x, y \in \mathbb{Z}^{\text{even}}\ \text{Even}(x + y)$.

Proof: *[Label where your proof starts.]*

Let $x$ and $y$ be arbitrary even integers. *[This is a prose form of universal instantiation.]*

By the definition of even, there exist integers $k$ and $p$ such that $x = 2k$ and $y = 2p$.

*[We know $x$ and $y$ are even, so we invoke the definition of even. Notice that because the definition of even involves an existential quantifier, we had to pick two different variable names.]*

By algebra, $x + y = 2k + 2p = 2(k + p)$. *[We are trying to make $x + y$ look like 2·(an integer)].*

Let $m = k + p$. $m \in \mathbb{Z}$ by the closure of the integers under addition. *[Must show that this is an integer.]*

$x + y = 2m$, and therefore $x + y$ is therefore even by the definition of even.

*[We have shown what we wanted to prove, that $x + y$ is even, so we may stop here. Some people would continue with the following:]*

Because $x$ and $y$ were chosen arbitrarily, we know $\forall x, y \in \mathbb{Z}^{\text{even}}\ \text{Even}(x + y)$.

*[The statement above is a prose form of universal generalization.]*

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Alternative restatement in symbols: $\forall x, y \in \mathbb{Z}\ (\text{Even}(x) \wedge \text{Even}(y)) \rightarrow \text{Even}(x + y)$.

Proof: *[Label where your proof starts.]*

Let $x$ and $y$ be arbitrary integers. *[This is a prose form of universal instantiation.]*

Assume $x$ and $y$ are even. *[This is a prose form of opening a conditional world.]*

By the definition of even, there exist integers $k$ and $p$ such that $x = 2k$ and $y = 2p$.

*[We know $x$ and $y$ are even, so we invoke the definition of even. Notice that because the definition of even involves an existential quantifier, we had to pick two different variable names.]*

By algebra, $x + y = 2k + 2p = 2(k + p)$. *[We are trying to make $x + y$ look like 2·(an integer)].*

$k + p \in \mathbb{Z}$ by the closure of the integers under addition. *[Must show that this is an integer.]*

$x + y$ is therefore even by the definition of even.

*[As in the first example, we have shown what we wanted to prove, that $x + y$ is even, so we may stop here. Some people would continue with language describing the closing of the conditional world, and the universal generalization.]*