Prove that if x is even, then $x + 5$ is odd.

Restatement in symbols: $\forall x \in \mathbb{Z}^{\text{even}}\ \text{Odd}(x + 5)$.

Proof: *[Label where your proof starts.]*

Let x be an arbitrary even integer. *[This is a prose form of universal instantiation.]*

By the definition of even, there exists an integer k such that $x = 2k$.
[We know x is even, so we invoke the definition of even.]

By algebra, $x + 5 = 2k + 5 = 2k + 4 + 1 = 2(k + 2) + 1$.
[We are trying to make $x + 5$ look like 2·(an integer)+1].

Let $m = k + 2$. $m \in \mathbb{Z}$ by the closure of the integers under addition. *[Must show that this is an integer.]*

$x + 5 = 2m + 1$, and therefore $x + 5$ is therefore odd by the definition of odd.

[We have shown what we wanted to prove, that $x + 5$ is even, so we may stop here. Some people would continue with the following:]

Because x was chosen arbitrarily, we know $\forall x \in \mathbb{Z}^{\text{even}}\ \text{Odd}(x + 5)$.

[The statement above is a prose form of universal generalization.]

Alternative restatement in symbols: $\forall x \in \mathbb{Z}\ \text{Even}(x) \to \text{Odd}(x + 5)$.

Proof: *[Label where your proof starts.]*

Let x be an arbitrary integer. *[This is a prose form of universal instantiation.]*

Assume x is even. *[This is a prose form of opening a conditional world.]*

By the definition of even, there exists an integer k such that $x = 2k$.
[We know x is even, so we invoke the definition of even.]

By algebra, $x + 5 = 2k + 5 = 2k + 4 + 1 = 2(k + 2) + 1$.
[We are trying to make $x + 5$ look like 2·(an integer)+1].

Let $m = k + 2$. $m \in \mathbb{Z}$ by the closure of the integers under addition. *[Must show that this is an integer.]*

$x + 5 = 2m + 1$, and therefore $x + 5$ is therefore odd by the definition of odd.

[As in the first example, we have shown what we wanted to prove, that $x + 5$ is odd, so we may stop here. Some people would continue with language describing the closing of the conditional world, and the universal generalization.]