Discrete Structures, Fall 2023, Homework 7

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Prove each of the following statements using "regular" or "weak" induction.

1. $\forall n \in \mathbb{Z}^{\geq 0} \ \sum_{i=0}^{n}(3i^2 - i) = n^2(n+1)$

2. Prove $\forall n \in \mathbb{Z}^{+} \ \prod_{i=1}^{n} i(i+1) = (n+1)(n!)^2$

   Hint: Recall that $n!$ denotes *the factorial of* n or n *factorial*, and is defined as $n! = n(n-1)(n-2)\cdots 2 \cdot 1$, with $0!$ defined to be 1. However, an alternate formula involving recursion is the following:

   $$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

   This recursive definition will be useful during the inductive step.

3. $\forall n \in \mathbb{Z}^{\geq 0} \ n(n+1)$ is even.

   Note: This is the same problem as question 1 on the last homework (prove that the product of any two consecutive integers is even). On that homework, you did this with the quotient-remainder theorem. On this homework, you should use induction (do not use the QRT here).

4. $\forall n \in \mathbb{Z}^{\geq 0} \ \ 5 \mid 7^n - 2^n$.

   Hint: Remember that $7^{k+1} = 7 \cdot 7^k$ and $2^{k+1} = 2 \cdot 2^k$. You can complete the inductive step of this proof in two different ways. The easier way involves manipulating the inductive hypothesis to get either 7^k or 2^k alone on one side of the equals sign, then substituting that into a piece of an equation in the inductive step.