

## Discrete Structures, Fall 2023, Homework 4

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Consider the following statement:

$$\exists x \in \mathbb{Z} \ x^2 = 4$$

Which of the following are correct translations of this statement? **Write down all the letters which are correct.** Note that some of the sentences below may be factually correct, but not correct translations of the original statement.

- (a) The square of each integer is 4.
  - (b) Some integers have squares of 4.
  - (c) The number  $x$ , when squared, is equal to 4, for some integer  $x$ .
  - (d) If  $x$  is an integer, then  $x^2 = 4$ . (Hint: read page 70.)
  - (e) Some integer has a square of 4.
  - (f) There is at least one integer whose square is 4.
2. Consider the following statement:

$$\forall n \in \mathbb{Z} \ \text{Even}(n) \rightarrow \text{Even}(n^2)$$

(You may interpret  $\text{Even}(x)$  to mean “ $x$  is an even number.”)

Which of the following are correct translations of this statement? **Write down all the letters which are correct.** Note that some of the sentences below may be factually correct, but not correct translations of the original statement.

- (a) All integers are even and have even squares.
- (b) Given any integer that is even, the square of that integer is also even.
- (c) For all integers, there are some whose square is even.
- (d) Any integer that is even has an even square.
- (e) If an integer is even, then its square is even.
- (f) All even integers have even squares.

3. Let  $E(x, y)$  mean “person  $x$  enjoys class  $y$ ,” let  $S$  be the set of all students, and let  $C$  be the set of all computer science classes.

Translate each of the following into English statements. Make your sentences as natural-sounding as possible, while still being precise in meaning.

- (a)  $\forall x \in S \forall y \in C E(x, y)$
- (b)  $\exists x \in S \exists y \in C E(x, y)$
- (c)  $\forall x \in S \exists y \in C E(x, y)$
- (d)  $\exists x \in S \forall y \in C E(x, y)$
- (e)  $\forall y \in C \exists x \in S E(x, y)$
- (f)  $\exists y \in C \forall x \in S E(x, y)$

4. Complete the following proofs using the method described in class (line numbers, rule justifications, etc).

- (a) P1:  $\exists w \in D \sim M(w) \rightarrow N(w)$   
P2:  $\forall x \in D \sim M(x) \vee R(x)$   
P3:  $\forall y \in D \sim N(y) \rightarrow \sim R(y)$   
Prove:  $\exists z \in D N(z)$
- (b) P1:  $\forall w \in D \sim R(w) \wedge Q(w)$   
P2:  $\forall x \in D Q(x) \rightarrow \sim(P(x) \wedge S(x))$   
P3:  $\forall y \in D (T(y) \rightarrow R(y)) \rightarrow P(y)$   
Prove:  $\forall z \in D S(z) \rightarrow T(z)$
- (c) P1:  $\forall w \in D \sim B(w)$   
P2:  $\forall x \in D Q(x) \rightarrow (R(x) \wedge T(x))$   
P3:  $\forall y \in D [B(y) \rightarrow \sim Q(y)] \rightarrow [R(y) \rightarrow B(y)]$   
Prove:  $\forall z \in D \sim Q(z)$

5. In this problem, you are given a number of statements in English about people and musical instruments. You are also given a number of statements in predicate logic. For each of the English statements, you must decide which predicate logic statements are true for the English statement in question.

Here are the predicate logic statements you can pick from:

Let  $L$  be the set of people “Kate, Lisa, John;” let  $M$  be the set of musical instruments “piano, trumpet, accordion;” and let the predicate  $P(x, y)$  mean “person  $x$  plays instrument  $y$ .”

1.  $\forall x \in L \exists y \in M P(x, y)$
2.  $\exists x \in L \forall y \in M P(x, y)$
3.  $\forall y \in M \exists x \in L P(x, y)$
4.  $\exists y \in M \forall x \in L P(x, y)$

You may assume that in each situation, each person plays only the instruments listed for him or her, and no others. In other words, if its not listed, they don't play it!

Here are the English statements. For each statement, write down the corresponding numbers of all the predicate logic statements above that are true for the English statement.

- (a) John plays piano, Kate plays trumpet, and Lisa plays accordian.
- (b) John plays piano, Kate plays piano and trumpet, and Lisa plays piano and accordian.
- (c) John plays trumpet, Kate plays piano, trumpet, and accordian, and Lisa doesn't play anything.
- (d) John plays trumpet, Kate plays piano and trumpet, and Lisa plays trumpet.
- (e) John plays trumpet, Kate doesn't play anything, and Lisa plays piano and accordian.
- (f) John plays accordian, Kate plays piano and accordian, and Lisa plays piano.
- (g) John plays piano, trumpet, and accordian, Kate plays trumpet and accordian, and Lisa plays accordian.
- (h) John plays piano and trumpet, Kate plays piano and accordian, and Lisa plays piano, trumpet, and accordian.