

Discrete Structures, Fall 2023, Homework 2

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Construct a complete truth table to help you determine if the following argument is valid or not. State whether it is valid or not, indicate the critical rows in the truth table, and explain why those rows support your answer.

Premise: $\sim q \vee (p \wedge \sim r)$

Premise: $\sim r \rightarrow p$

Conclusion: $q \rightarrow r$

2. For each of the following, if a *single*, valid rule of inference can lead from the given premises to the given conclusion, state what rule of inference would be used. If no valid rule could be used, write “no rule.” Note that you must still write “no rule” even if the conclusion follows from the premises but not in one step.

(a) Premise: $(m \rightarrow p) \vee (n \rightarrow p)$

Premise: n

Conclusion: p

(b) Premise: $(a \wedge b) \rightarrow (z \vee y)$

Premise: $\sim(z \vee y)$

Conclusion: $\sim(a \wedge b)$

(c) Premise: $r \wedge (q \vee p)$

Premise: $\sim q$

Conclusion: p

(d) Premise: $k \wedge m$

Conclusion: $(k \wedge m) \vee (k \rightarrow (n \rightarrow m))$

(e) Premise: $r \wedge (q \vee p)$

Premise: $\sim q$

Conclusion: $r \wedge p$

(f) Premise: $g \rightarrow h$

Premise: $(e \wedge k) \vee g$

Premise: $(e \wedge k) \rightarrow h$

Conclusion: h

3. Complete the following proofs using the framework discussed in class. Each line of your proof must be justified with a rule of inference or logical equivalence and appropriate line numbers.

(a) P1 $\sim p$
P2 $(q \wedge \sim p) \rightarrow m$
P3 $\sim r \vee q$
P4 r

Prove: m

(b) P1 $\sim k \wedge g$
P2 $m \rightarrow \sim g$
P3 $(\sim f \rightarrow k) \vee m$

Prove: f