

Discrete Structures, Fall 2023, Homework 1

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Convert the following sentences to logical statements using symbols assuming that “ p ”, “ b ” and “ m ” represent the propositions below.

p = “Morgan is taking a physics class.”

b = “Morgan is taking a biology class.”

m = “Morgan is taking a math class.”

- (a) Morgan is taking a biology class and a math class, but not a physics class.
 - (b) Morgan is taking a physics class, and either a biology or math class (but not both bio and math).
 - (c) Morgan is taking a physics class, and either a biology or math class (perhaps both bio and math).
 - (d) If Morgan is taking a biology class, then they are also taking a math class, but if Morgan is not taking a biology class, then they are taking a physics class.
2. For each of the sentences below, determine if the sentence is a statement. If the sentence is a statement, tell whether it is true or false.
 - (a) If a tree falls in the forest and no one is around to hear it, does it make a sound?
 - (b) If $2 + 2 = 5$, then I am the very model of a modern major-general.
 - (c) This statement refers to itself.
 - (d) This statement is false.
 3. Express the **negations** of the following statements in normal English sentences.
 - (a) Sally is a computer science major and Sally’s brother is a math major.
 - (b) Either the professor is late or my watch is fast.
 4. For each of the following statements, give the contrapositive, converse, and inverse statements (label them) in normal English, using the phrasing “If . . . , then . . .” You may change verb tenses to improve the grammar. **Do not** use “if” or “only if” in the middle of your sentences.
 - (a) “If you conquer yourself, then you conquer the world.”¹
 - (b) I will be able to retire if I save enough money.
 - (c) You can go to the party only if you get good grades.
 5. Let x , y and z be statements. Construct a complete truth table (meaning don’t leave out any columns) for the statement $(x \rightarrow \sim y) \wedge (\sim x \vee z)$.
 6. Are the statements $\sim(p \vee q)$ and $\sim p \vee \sim q$ logically equivalent? Use a complete truth table to justify your answer, and explain (in English) why the truth table supports your answer.

¹From *Aleph*, by Paulo Coelho.