Running time of algorithms

How can we measure the running time of algorithms?

- Idea: Use a stopwatch.
  - What if we run the algorithm on a different computer?
  - What if we code the algorithm in a different programming language?
  - What if the computer is doing other things in the background while timing our algorithm?
  - Timing the algorithm doesn’t (directly) tell us how it will perform in other cases besides the ones we test it on.

How can we measure the running time of algorithms?

- Idea: Count the number of “basic operations” in an algorithm.
  - “Basic operations” are things the computer can do “in a single step,” like
    - Printing a single value (number or string)
    - Comparing two values
    - (simple) math, like adding, multiplying, powers
    - Assigning a variable a value

- How many basic operations are done in this algorithm?
  - Only count printing as a basic operation.

```
// assume array is an array of three ints
for (int i = 0; i < 3; i++) {
   System.out.println(array[i]);
}
// assume array2 is an array of six ints
for (int i = 0; i < 6; i++) {
   System.out.println(array2[i]);
}
```

- How many basic operations are done in this algorithm?
  - Only count printing as a basic operation.

```
// assume array is an array of ints
for (int i = 0; i < array.length; i++) {
    System.out.println(array[i]);
}
```

If n = array3.length, what is a general formula for how long this algorithm takes, in terms of n?

- How many basic operations are done in this algorithm, ***in the worst possible case***?
  - Only count printing as a basic operation.

```
// assume array is an array of ints
for (int i = 0; i < array.length; i++) {
    if (array[i] > 10) {
        System.out.println(array[i]);
    }
}
```

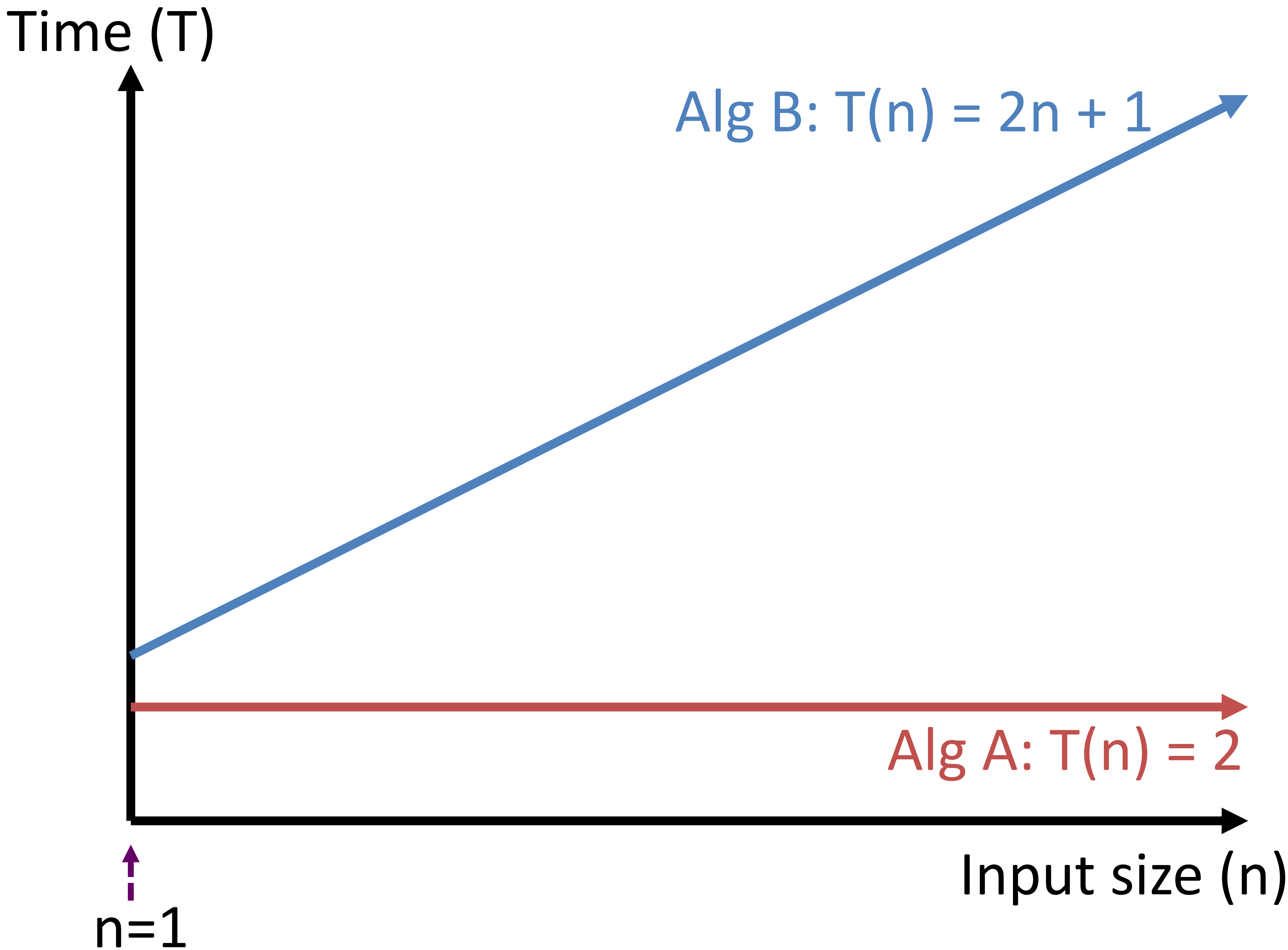
If n = array.length, what is a general formula for how long this algorithm takes, in terms of n, in the worst case?

- Computer scientists often consider the running time for an algorithm in the ***worst case***, since we know the algorithm will never be slower than that.
  - Sometimes we also care about ***average*** running time.
- We express the running time of an algorithm as a function in terms of "*n*," which represents the size of the input to the algorithm.
- For an algorithm that processes an array or arraylist, *n* is the length of the array or arraylist.

```java
/* Assume for both algorithms, var and n are
   already defined as positive integers.
   Basic ops are printing and adding. */

// algorithm A
var = var + n;
System.out.println(var);

// algorithm B
for (int i = 0; i < n; i++) {
   var++;
}
System.out.println(var);
```



Time (T)
Alg B: T(n) = 2n + 1
Alg A: T(n) = 2
Input size (n)
n=1

- We group running times together based on how they grow as *n* gets really big.
- If the running time stays exactly the same as *n* gets big (*n* has no effect on the algorithm's speed), we say the running time is **constant**.
- If the running time grows proportionally to *n*, we say the running time is **linear**.
  – If the input size doubles, the running time roughly doubles.
  – If the input size triples, the running time roughly triples.

```java
// algorithm A
var = var + n;
System.out.println(var);
```

What class does algorithm A fall into? [constant or linear]

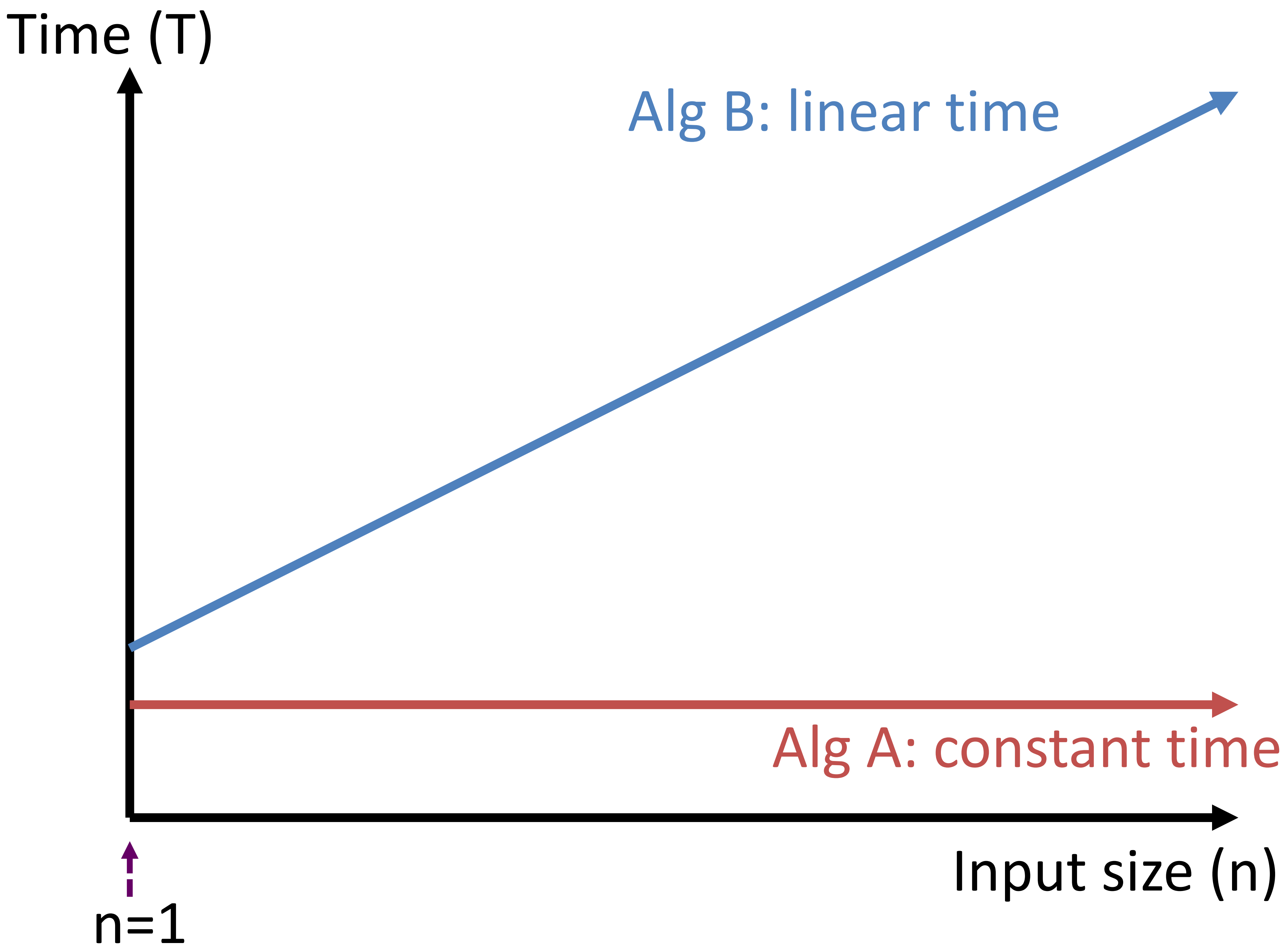
```java
// algorithm B
for (int i = 0; i < n; x++) {
   var++;
}
System.out.println(var);
```

What class does algorithm B fall into? [constant or linear]

Which is "better?"

- In general, we prefer algorithms that run faster.
  - That is, as the algorithm's input size grows, the time required to run the algorithm should grow as slowly as possible.

- Therefore, an algorithm that runs in constant time is *usually* preferred over a linear-time algorithm.

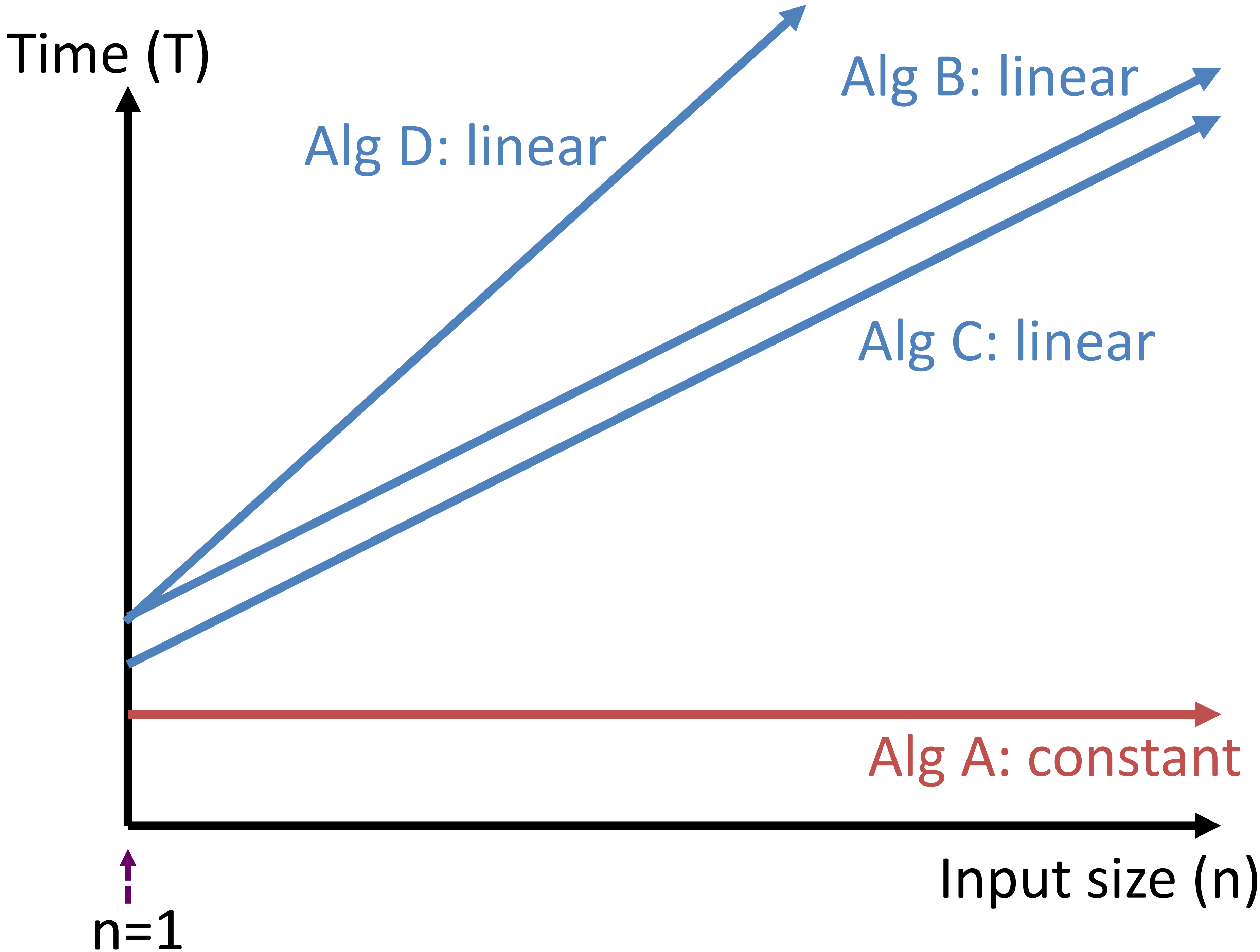

Time (T)
Alg B: linear time
Alg A: constant time
Input size (n)
n=1

```
// Calculate formulas for basic
// operations (adding, printing).

// algorithm C:
// assume array has n ints in it
int sum = 0;
for (int i = 0; i < array.length; i++) {
    sum += array[i];
}
```

```
// Calculate formulas for basic
// operations (adding, printing).

// algorithm D:
// assume array has n ints in it
int sum = 0;
for (int i = 0; i < array.length; i++) {
    if (array[i] > 10) {
       sum += array[i];
    }
    System.out.println(sum);
}
```


Time (T)
Alg D: linear
Alg B: linear
Alg C: linear
Alg A: constant
Input size (n)
n=1

Categories have special names, which use ***big-O notation.***

Constant time algorithm: **O(1)**

Read as “big-oh of 1” or “oh of 1”

Linear time algorithm: **O(n)**

Read as “big oh of n” or “oh of n”

These categories give us a rough estimate of the "order of growth" of an algorithm = how the run time changes as we increase the input size, without worrying about details.

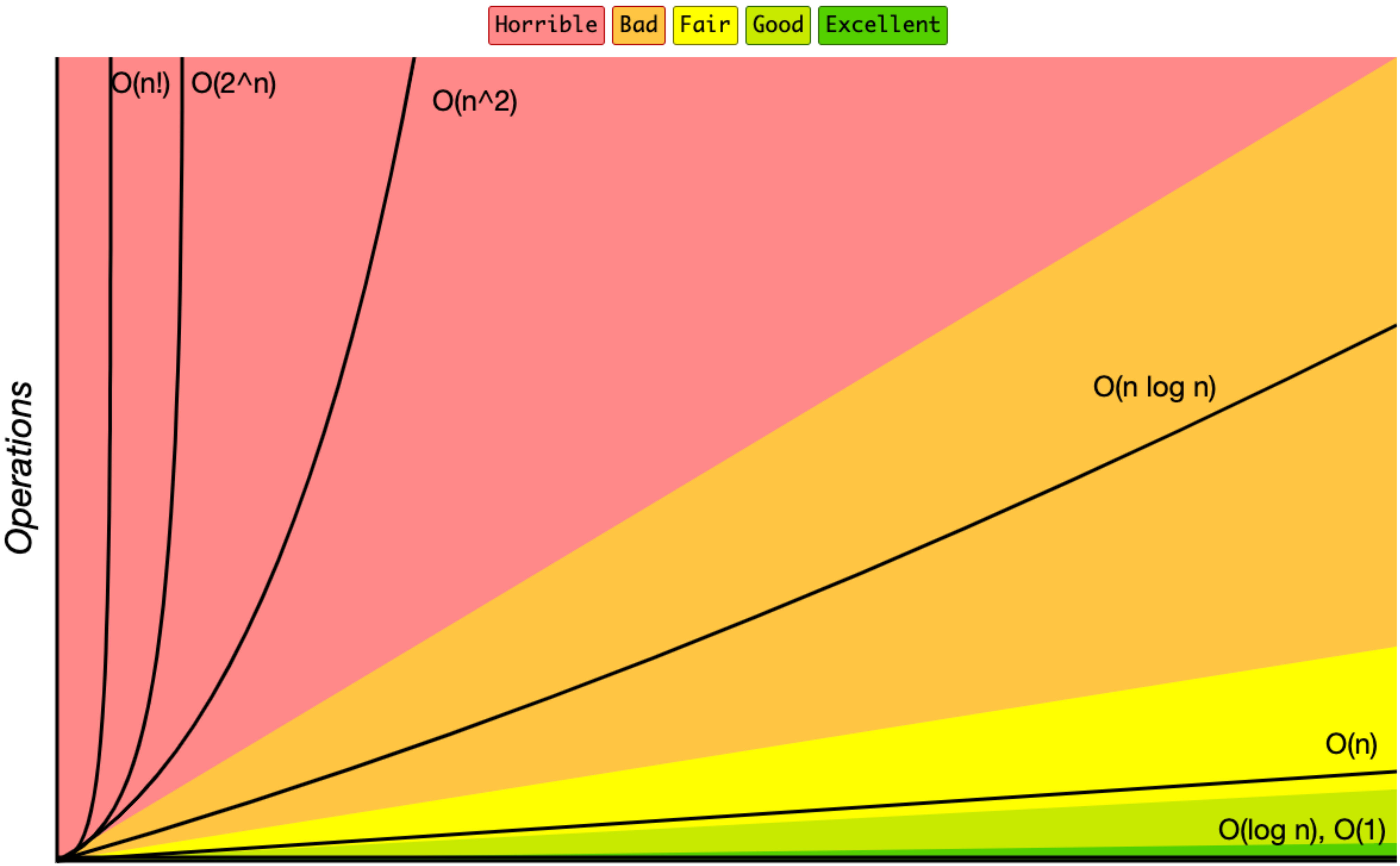
Count (or estimate) the basic operations.

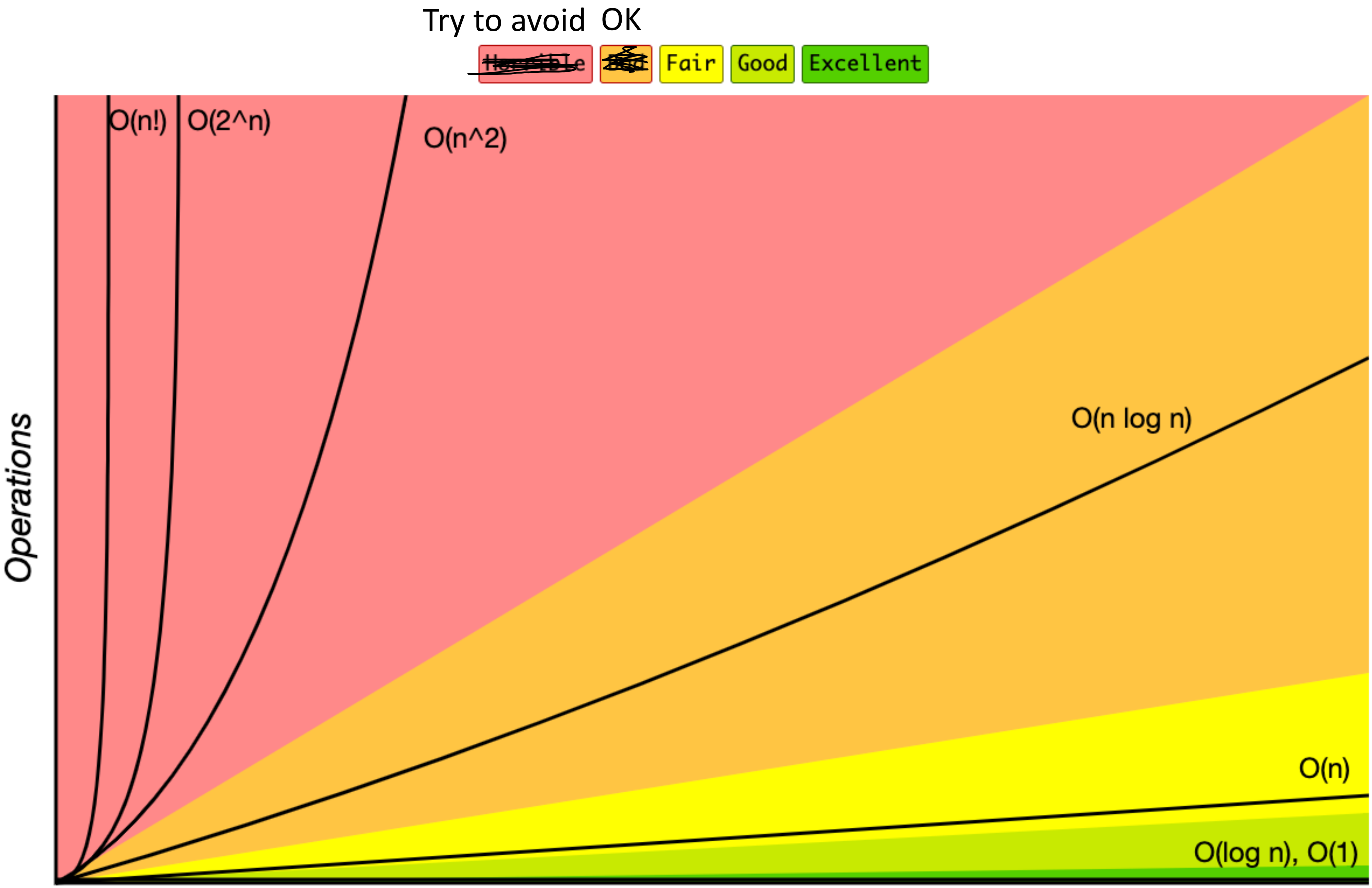
Assume n = array.length.

```
int sum = 0;
for (int row = 0; row < array.length; row++)
{
  for (int col = 0; col < array[row].length; col++)
  {
    sum += array[row][col];
  }
}
```

- An algorithm which doesn't get slower as input size increases is a **constant-time** algorithm.
- An algorithm whose running time grows proportionally to input size is a **linear-time** algorithm.
- An algorithm whose running time grows proportionally to the square of the input size is a **quadratic-time** algorithm.
  - $O(n^2)$
- What about binary search?

- Some problems have algorithms that run even more slowly than quadratic time.
  - Cubic time (n^3), higher polynomials, …
  - Exponential time (2^n) is even slower! [Fibonacci]
- In some situations, we ***depend*** on the fact that we don't have fast algorithms to solve problems.
  - For instance, many cryptographic algorithms that enable us to send passwords securely over the internet depend on us not having fast algorithms to break the encryption.


Horrible
Bad
Fair
Good
Excellent
O(n!)
O(2^n)
O(n^2)
O(n log n)
O(n)
O(log n), O(1)
Operations


Try to avoid
OK
Fair
Good
Excellent
O(n!)
O(2^n)
O(n^2)
O(n log n)
O(n)
O(log n), O(1)
Operations

One million “basic” operations per second.

	logarithmic	linear	quadratic	exponential
n = 10				
n = 20				
n = 30				
n = 50				
n = 100				
n = 1,000				
n = 10,000				
n = 100,000				
n = 1,000,000				

One million “basic” operations per second.

	logarithmic	linear	quadratic	exponential
n = 10	0.0033 ms			
n = 20	0.0043 ms			
n = 30	0.0049 ms			
n = 50	0.0056 ms			
n = 100	0.0066 ms			
n = 1,000	0.0099 ms			
n = 10,000	0.0133 ms			
n = 100,000	0.0166 ms			
n = 1,000,000	0.0199 ms			

One million "basic" operations per second.

	logarithmic	linear	quadratic	exponential
n = 10	0.0033 ms	0.01 ms		
n = 20	0.0043 ms	0.02 ms		
n = 30	0.0049 ms	0.03 ms		
n = 50	0.0056 ms	0.05 ms		
n = 100	0.0066 ms	0.1 ms		
n = 1,000	0.0099 ms	1 ms		
n = 10,000	0.0133 ms	10 ms		
n = 100,000	0.0166 ms	0.1 sec		
n = 1,000,000	0.0199 ms	1 sec		

One million “basic” operations per second.

	logarithmic	linear	quadratic	exponential
n = 10	0.0033 ms	0.01 ms	0.1 ms	
n = 20	0.0043 ms	0.02 ms	0.4 ms	
n = 30	0.0049 ms	0.03 ms	0.9 ms	
n = 50	0.0056 ms	0.05 ms	2.5 ms	
n = 100	0.0066 ms	0.1 ms	0.01 sec	
n = 1,000	0.0099 ms	1 ms	1 sec	
n = 10,000	0.0133 ms	10 ms	1.67 min	
n = 100,000	0.0166 ms	0.1 sec	2.77 hours	
n = 1,000,000	0.0199 ms	1 sec	11.57 days	

One million “basic” operations per second.

	logarithmic	linear	quadratic	exponential
n = 10	0.0033 ms	0.01 ms	0.1 ms	1.024 ms
n = 20	0.0043 ms	0.02 ms	0.4 ms	
n = 30	0.0049 ms	0.03 ms	0.9 ms	
n = 50	0.0056 ms	0.05 ms	2.5 ms	
n = 100	0.0066 ms	0.1 ms	0.01 sec	
n = 1,000	0.0099 ms	1 ms	1 sec	
n = 10,000	0.0133 ms	10 ms	1.67 min	
n = 100,000	0.0166 ms	0.1 sec	2.77 hours	
n = 1,000,000	0.0199 ms	1 sec	11.57 days	

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n = 50	0.0056 ms	0.05 ms	2.5 ms	
n = 100	0.0066 ms	0.1 ms	0.01 sec	
n = 1,000	0.0099 ms	1 ms	1 sec	
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n = 30	0.0049 ms	0.03 ms	0.9 ms	17.9 min
n = 50	0.0056 ms	0.05 ms	2.5 ms	
n = 100	0.0066 ms	0.1 ms	0.01 sec	
n = 1,000	0.0099 ms	1 ms	1 sec	
n = 10,000	0.0133 ms	10 ms	1.67 min	
n = 100,000	0.0166 ms	0.1 sec	2.77 hours	
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n = 30	0.0049 ms	0.03 ms	0.9 ms	17.9 min
n = 50	0.0056 ms	0.05 ms	2.5 ms	35.7 years
n = 100	0.0066 ms	0.1 ms	0.01 sec	
n = 1,000	0.0099 ms	1 ms	1 sec	
n = 10,000	0.0133 ms	10 ms	1.67 min	
n = 100,000	0.0166 ms	0.1 sec	2.77 hours	
n = 1,000,000	0.0199 ms	1 sec	11.57 days	

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n = 20	0.0043 ms	0.02 ms	0.4 ms	1.049 sec
n = 30	0.0049 ms	0.03 ms	0.9 ms	17.9 min
n = 50	0.0056 ms	0.05 ms	2.5 ms	35.7 years
n = 100	0.0066 ms	0.1 ms	0.01 sec	4×10^{16} years
n = 1,000	0.0099 ms	1 ms	1 sec	3×10^{287} years
n = 10,000	0.0133 ms	10 ms	1.67 min	----
n = 100,000	0.0166 ms	0.1 sec	2.77 hours	----
n = 1,000,000	0.0199 ms	1 sec	11.57 days	----